ARULMIGU PALANIANDAVAR ARTS COLLEGE FOR WOMEN, PALANI

DEPARTMENT OF MATHEMATICS

Learning Resources

Title of the paper: LINEAR ALGEBRA

Prepared By

Dr.K.Meena, Associate Professor and Head

ONE MARK QUESTIONS AND ANSWERS

UNIT-I

1. **For each F and vϵV we have an element αvϵV This operation is called ---- Ans: (scalar multiplication)**
2. **Any field K is a vector space over any of its ----- Ans: (sub field)**
3. **Let V is a vector space over F .A non-empty subset W of V is a subspace of V iff w_1, w_2 ϵ W & α,$\beta\epsilon$F ==>--------- Ans: (αu+βvϵW)**
4. **The intersection of two subspace of a vector space is a ------- Ans: (subspace)**
5. **The union of two subspace of a vector space----------------**
   Ans: (need not be a subspace)
6. **The union of two subspace of a vector space is a subspace iff -----**
   Ans: (one is contained in the other)
7. **Let U and V are vector space over F then the mapping T of U onto V is said to be homomorphism if -------a) T(u+v)=T(u) + T(v) . b) T(αu)= α T(u) . c)T(αu+βv) = α T(u)+β T(v). d) T(α+β) =T α) +T(β)**
   Where u ,v ϵV and α ,β ϵF . Ans: (c)T(αu+βv) = α T(u)+β T(v))
8. **A homomorphism T of vector spaces is also called -----**
   Ans: (a linear transformation)
9. **A homomorphism T of vector spaces if T is 1-1. Then T is called ----**
   Ans: (monomorphism)
10. **A homomorphism T of vector spaces if T is 1-1 and onto , Then T is called ----------- Ans: (isomorphism)**
11. **A homomorphism T of vector spaces if T is onto, Then T is called ----**
    Ans: (epimorphism)

12. **A linear transformation T : V --->F is called----- Ans: (linear functional)**
13. **Let V and W vector spaces over a field F and T : V --->W be a linear transformation , Then kernel of T =------- Ans: ({v/v ϵV & T(v) =0})**
14. **Let T : V --->W be a linear transformation Then T is a monomorphism iff --------- Ans: (ker T = {0})**
15. **Let A and B be subspaces of a vector spaces V. Then V is called the direct sum of A and B if ------ Ans: (A+B=V & A∩B ={0})**
16. **If A and B be subspaces vector spaces of V. Then A+B/B is isomorphic to ---------------- Ans: (A/A∩B)**
17. **In the linear transformation T : V_3(R) ---→ V_3(R) defined by T(x,y,z) =(x,y,0) . Then ker T =------ Ans: ({(0,0,z)/zϵ R})**

Say True or False

18. **Any vector space is an abelian group with respect to vector addition. Ans:True.**
19. **C is vector space over the field R. Ans:True.**
20. **R is not a vector space over C. Ans:True.**

UNIT-II

21. **Any vector space contains an infinite number of elements. Ans: False**
22. **R[x] is a vector space over C. Ans: False.**
23. **The set of all polynomials of even degree in R[x] is a vector space over R -**
 Ans: False
24. **In V_2(R) Let S={(1,0), (0,1)} Then L(S)=-------a) S b) {(x,0)/xϵ R}**
 c) {(0,y)/yϵ R} d) V_2(R) Ans: d) V_2(R)
25. **In V_2(R) Let S={(4,0)} Then L(S)=-------a) S b) {(x,0)/xϵ R} c) {(0,y)/yϵ R}**
 d) V_2(R) Ans: b) {(x,0)/xϵ R}
26. **In V_3(R) Let S = {e_1, e_2, e_3} Then L(S)=------------------**
 a) S b) {(x,y,0)/x,y ϵ R} c) {(0,y,z)/y,z ϵ R} d) V_3(R) Ans: V_3(R)
27. **In R[x] Let S= {1,x,x^2,x^3} Then L(S)=-------**
 a) R[x] b) V_3(R) c) S d) set of all polynomials of degree ≤ 3
 Ans: d) set of all polynomials of degree ≤ 3
28. **In R Let S={1} Then L(S) a) S b) C c) R d) Q Ans: c) R**
29. **dim M_2(R) =-----------**
 a) 1 b) 2 c)3 d)4 Ans: d)4

30. In $V_3(R)$ Let S =L[{(1,1,1)}] and T=[{(-1,-1,-1)}] Then dim(S∩T) is --------

a) 1 b) 0 c) 2 d) none Ans: a) 1

31. Any two bases of a finite dimensional vector space V have ------
Ans: (the same number of elements)

32. Let V be a vector space of dimension n. Then any set of m vectors where
m>n is ------------ Ans: (linearly dependent)

33. Let V be a vector space of dimension n. Then any set of m vectors where
m<n is -------------- Ans: (cannot span V)

Say True or False

34. Given any field F there exists a vector space of dimension n over F . Ans: True.

35. Any two vector space over a field F are isomorphic iff they have the same
dimension- Ans: True.

36. Any two bases of a finite have the same number of elements . Ans:True

37. Any vector space of dimension n>1 has non trivial subspace. Ans:True

38. If Vis vector space of dimension n and m<n then there exists a subspace of V of dimension m. Ans:True

39. Any linear transformation from a finite dimensional vector space V to finite dimensional vector space W can be represented by a metric. Ans:True

40. If V is a finite dimensional vector space and if $u_1,u_2,\ldots u_m$ span V then some subset of $u_1,u_2,\ldots u_m$ form a ------- Ans: (basic of V)

UNIT-III

41. If $u_1,u_2,\ldots u_m$ is a basic of V over F and if $w_1,w_2,\ldots w_m$ in V are linearly independent over F then------ Ans: (m≤n)

42. If A and B are finite dimensional subspaces of a vector space V then A+B is Finite dimensional and dim(A+B) =------ Ans: (dim(A)+dim(B)-dim(A∩B))

43. If V=A+B Then dim V =------ Ans: (dimA +dimB)

44. Hom(V,W) to be the set of all vector space--------of V into W (Ans: homomorphism)

45. If V and W are of dimension m and n,respectively over F then Hom(V,W) is of dimension --------over F. (Ans: mn)

46. If dim V =m then dim Hom(V,V) =-----(Ans: m^2)

47. If dim $_F$V =m then dim$_F$ Hom(V,F) =-----(Ans: m)

48. If V is vector space then its dual space is --------(Ans: Hom(V,F))

49. An element of the dual space of V will be called a ------ (Ans: linear functional on V into F)

50. Let V be a finite dimensional vector space over F . Let $\{v_1,v_2,\ldots,v_n\}$ be a basis of V Let $v_1,v_2,\ldots,v_n$ be the functional defined by $V_i(V_j)$ ={1 if i=j &0 if i≠j Then $\{ v_1,v_2,\ldots,v_n \}$ is a basis of V Ans: (This known as Basis of dual space)

51. If V is finite dimensional and v≠0ϵV then there is an element f ϵV such that ---------------(Ans: f(v) ≠0)

52. If V is finite dimensional , Then Ψ is an isomorphism of V onto V Ans: (True)

53. Let V be a finite dimensional vector space over a field F . Then Ψ : V----> V defined by $\Psi(v) = T_v$ is an ------------- where T_v (f) =f(v) for all f ϵ V Ans: (Isomorphism)

54. If W is a subspace of V Then the annihilator of W ,A(W) = -------------- Ans: { f ϵ V/ f(w) =0 for all w ϵ W}

55. A(A(W)) =-------------Ans: W

56. If V is finite dimensional and W is a subspace of V, Then W is isomorphic to V/A(W) and dimA(W) = --------- Ans: dim V –dimW

57. A(W) is a subspace of V Ans:True

58. A vector space V is called a real vector space or----------- Ans: (complex vector space)

59. Two vectors v= (x_1, x_2, x_3) and w= (y_1, y_2, y_3) v.w=--------- Ans: $(x_1 y_1, x_2 y_2, x_3 y_3)$

60. Two vectors V& W are given Find the length of v is --------- Ans: $\sqrt{v}.v$

UNIT-IV

61. Two vectors V& W are given Find the angle θ between v & w is ------ Ans: cosθ=v.w/ $\sqrt{v}.v\sqrt{w}.w$

62. <u, αv+βw> =-------- Ans: α<u, v> + β<u, w>

63. If vϵ V then the norm of v is ------- Ans: $\| v \| = \sqrt{< v, v >}$

64. In norm of vector space V, vϵ V is called unit vector if ------- Ans: || v || = 1

65. Write Schwarz inequality. Ans: Let V be an inner product space and Let u, v ϵ V Then |<u,v>| ≤ ||u|| ||v|| .

66. Write Triangle inequality. Ans: For any two vectors u, v in V || u + v || ≤ ||u|| + ||v||

67. Let V be an inner product space and let u, v ϵ V, Then u is said to be orthogonal to v if ------------- Ans: < u, v > = 0

68. Let V be an inner product space . A set $S = \{v_i\}$ of vectors in V is said to be an orthonormal set if -------------
Ans: i) $\langle v_i, v_i \rangle = 1$ for each i ii) $\langle v_i, v_j \rangle = 0$ for each $i \neq j$

69. Every finite dimensional inner product space has an ------------------
Ans: orthonormal basis

70. Let V be an inner product space & W a subspace of V .Then the orthogonal complement of W is $W^{\perp}$ =----- Ans: $W^{\perp} = \{v \in V / \langle v, w \rangle = 0 \text{ for all } w \in W\}$

71. In an inner product space V $\| u+v \|^2 + \| u-v \|^2$ =----------- Ans: $2(\|u\|^2 + \|v\|^2)$

72. Find the norm of (1, 3, -5) in R^3 with standard inner product. Ans : $\sqrt{35}$

73. Find the norm of (2, 0, -1) in R^3 with standard inner product. Ans : $\sqrt{5}$

74. Find the norm of (1, 2, 3) in V3(R) with standard inner product. Ans : $\sqrt{14}$

75. $\|\alpha v\|$ =-------------- Ans: $|\alpha| \, \|v\|$

76. $\langle \alpha u+\beta v, \alpha u+\beta v \rangle$ =----------------Ans: $\alpha\alpha\langle u, u\rangle + \alpha\beta\langle u, v\rangle + \alpha\beta\langle v, u\rangle + \beta\beta\langle v, v\rangle$

77. If a, b, c are real numbers such that a>0 and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers λ then ------------ Ans: $b^2 \leq ac$

78. Let V be a finite dimensional inner product space and W a subspace of V. Then $(W^{\perp})^{\perp}$ =--------------- Ans: W
Say True or False

79. Say True or False $\|x+y\| = \|x\| + \|y\|$ Ans: False

80. Say True or False $S^{\perp}$ is a subspace of V only when S is a subspace.
Ans: False

UNIT -V

81. If A is any 3x2 matrix then $(A^T)^T$ is ----- Ans: 3x2 matrix

82. If A=$\begin{pmatrix} 1+i & 2 \\ 2-i & i \end{pmatrix}$ then the conjugate of A is ------- Ans: $\begin{pmatrix} 1-i & 2 \\ 2+i & -i \end{pmatrix}$

83. Any skew Hermitian matrix with entries from real numbers is always.......
Ans: Skew symmetric matrix

84. A square matrix A is said to be idempotent if -------Ans: A^2=A

85. A square matrix A is said to be involutory if -------Ans: A^2=I

86. If A=$\begin{pmatrix} 1 & 2 \\ 3 & n \end{pmatrix}$ and B= $\begin{pmatrix} m & 2 \\ n & 4 \end{pmatrix}$ are singular matrices then the value of mn is------ Ans: 3

87. If A= $\begin{pmatrix} 1 & 0 & 0 \\ 2 & x & 4 \\ 4 & 1 & x \end{pmatrix}$ is a singular matrix then the values of x are ------
Ans: 2,-2

88. The characteristic polynomial of A=$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ is -----Ans: $x^2 - 2x + 5$

89. If the Eigen values of A are -1, 2, 5 the Eigen values of $(A^2)^{-1}$ are----

Ans: 1, $\frac{1}{4}, \frac{1}{25}$

90. If the eigen values of A are -1, 2, 5 the eigen values of $(5A)^{-1}$ are----

Ans: $\frac{-1}{5}, \frac{1}{10}, \frac{1}{25}$

91. The product of the eigen value of $\begin{pmatrix} -3 & 3 \\ -2 & 4 \end{pmatrix}$ is ----Ans: -6

92. The sum of the eigen values of $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$ is ----- Ans: 0

93.The eigen values of the matrix I_2 are ---- Ans: 1, 1

94. The characteristic polynomial of I_2 is ----- Ans: x^2-2x+1

95.If A=$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then adjA =----- Ans $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

96. If A=$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then A^{-1}=---- Ans: $\begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$

97. The matrix of the quadratic form $2x^2 - 4xy + 3y^2$ is ---Ans: $\begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}$

98. The quadratic form of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is ----- Ans : $x^2 + y^2$

99. The quadratic form of the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is ----- Ans : $2x_1x_2$

100. The matrix of the quadratic form $ax_1^2 + ax_2^2 - 2bx_1x_2$ is ----

Ans: $\begin{pmatrix} a & -b \\ -b & a \end{pmatrix}$

Staff in charge

Gram-Schmidt Orthogonalisation Process.

Every finite dimensional inner product space has an orthonormal basis.

~~Proof~~. **Examples.**

Apply Gram Schmidt Process to construct an orthonormal basis for $V_3(R)$. with the Standard inner product with the following bases. a) $(1,-1,0)$ $(2,-1,-2)$ $(1,-1,-2)$.

Let $v_1 = (1,-1,0)$ $v_2 = (2,-1,-2)$ $v_3 = (1,-1,-2)$.

We know that $v_1 = (a, b, c)$ & $v_2 = (a_1, b_1, c_1)$

$$\langle v_1, v_2 \rangle = aa_1 + bb_1 + cc_1.$$

Take $w_1 = v_1 = (1,-1,0)$.

then $\|w_1\|^2 = \langle w_1, w_2 \rangle = 1\times1 + (-1)(-1) + 0\times0$
$= 1+1 = 2$ $\therefore \boxed{\langle w_1, w_2 \rangle = 2}$

$\therefore \|w_1\|^2 = 2 \Rightarrow \|w_1\| = +\sqrt{2}$.

Let $w_2 = v_2 - \dfrac{\langle v_2, w_1 \rangle}{\|w_1\|^2} w_1$.

$\langle v_2, w_1 \rangle = 2\times1 + (-1)(-1) + (-2)\times0$ $\quad (\because v_2 = (2,-1,-2)$, $w_1 = v_1 = (1,-1,0))$.

$\langle v_2, w_1 \rangle = 2+1 = 3$.

$\therefore w_2 = (2,-1,-2) - \dfrac{3}{2}(1,-1,0)$.

$= (2,-1,-2) - (3/2, -3/2, 0)$.

$= (2 - 3/2, -1 + 3/2, -2 - 0)$

$w_2 = \left(\dfrac{1}{2}, \dfrac{1}{2}, -2\right)$ $\quad \|w_2\| = \dfrac{3}{\sqrt{2}}$

$\|w_2\|^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (-2)^2 = \frac{1}{4} + \frac{1}{4} + 4 = \frac{1+1+16}{4} = \frac{18}{4} = \frac{9}{\ }$

Let $w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\|w_1\|^2} w_1 - \frac{\langle v_3, w_2 \rangle}{\|w_2\|^2} w_2$

$v_3 = (1, -1, -2)$

$w_1 = v_1 = (1, -1, 0)$

$w_2 = (1/2, 1/2, -2)$.

$\langle v_3, w_1 \rangle = 1 \times 1 + (-1)(-1) + (-2 \times 0).$

$= 1 + 1 = 2.$

$\langle v_3, w_2 \rangle = 1 \times 1/2 + (-1)(\frac{1}{2}) + (-2)(-2)$

$= \frac{1}{2} - \frac{1}{2} + 4 = 4.$

$\therefore w_3 = (1, -1, -2) - \frac{2}{2}(1, -1, 0) - \frac{4}{(\frac{9}{2})}(1/2, 1/2, -2).$

$= (1, -1, -2) - (1, -1, 0) - \frac{8}{9}(1/2, 1/2, -2).$

$= \left(1 - 1 - \frac{8}{9} \times \frac{1}{2},\ -1 + 1 - \frac{8}{9}(\frac{1}{2}),\ -2 - 0 + \frac{8}{9}(2)\right)$

$w_3 = \left(\frac{-4}{9}, \frac{-4}{9}, -\frac{2}{9}\right).$

$\|w_3\|^2 = (\frac{-4}{9})^2 + (\frac{-4}{9})^2 + (\frac{-2}{9})^2 = \frac{16 + 16 + 4}{9^2} = \frac{36}{9^2} = (\frac{6}{9})^2.$

$\|w_3\| = \frac{6}{9}.$

The orthogonal basis is $\{w_1, w_2, w_3\}$.

Hence the orthonormal basis is

$\left\{ \frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}, \frac{w_3}{\|w_3\|} \right\}.$

$\therefore \left\{ \frac{(1, -1, 0)}{\sqrt{2}}, \frac{(1/2, 1/2, -2)}{3/\sqrt{2}}, \frac{(\frac{-4}{9}, \frac{-4}{9}, \frac{-2}{9})}{\frac{6}{9}} \right\}$

$= \left\{ (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (\frac{\frac{1}{2}}{3/\sqrt{2}}, \frac{1/2}{3/\sqrt{2}}, -\frac{2}{3/\sqrt{2}}), (\frac{-4/9}{6/9}, \frac{-4/9}{6/9}, \frac{-2/9}{6/9}) \right\}$

$= \left\{ (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (\frac{\sqrt{2}}{6}, \frac{\sqrt{2}}{6}, -\frac{2\sqrt{2}}{3}), (\frac{-2}{3}, \frac{-2}{3}, -\frac{1}{3}) \right\}$

K Meena

Staff in charge